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Tugas Analisis Vektor (c)

1). Diketahui fungsi vektor $F(x,y) = 2xy^3\hat{i} + (1+3x^2y^2)\hat{j}$

a). Buktikan bahwa $F(x,y)$ adalah medan vektor konservatif pada bidang x, y

Jawab :

akan dibuktikan $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$$F(x,y) = \underbrace{2xy^3}_{M}\hat{i} + \underbrace{(1+3x^2y^2)}_{N}\hat{j}$$

$$\frac{\partial M}{\partial y} = 6xy^2 \qquad \frac{\partial N}{\partial x} = 6xy^2$$

∴ TERBUKTI bahwa $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Sehingga fungsi vektor $F(x,y) = 2xy^3\hat{i} + (1+3x^2y^2)\hat{j}$ merupakan medan vektor konservatif

b). Jika gaya $F(x,y)$ adalah konservatif maka $F = \nabla\phi$ dan $\phi(x,y)$ disebut potensial skalar. Dapatkan fungsi $\phi(x,y)$

Jawab :

$$F = \nabla\phi$$
$$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j}\right)\phi$$
$$= \left(\frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j}\right)$$

> $\frac{\partial\phi}{\partial x} = 2xy^3$ > $\frac{\partial\phi}{\partial y} = 1+3x^2y^2$

$\int \partial\phi = \int 2xy^3\,\partial x$ $\int \partial\phi = \int (1+3x^2y^2)\,\partial y$

$\phi = x^2y^3 + C_1$ ... (i) $\phi = y + x^2y^3 + C_2$ ... (ii)

Substitusi (i) ke (ii)

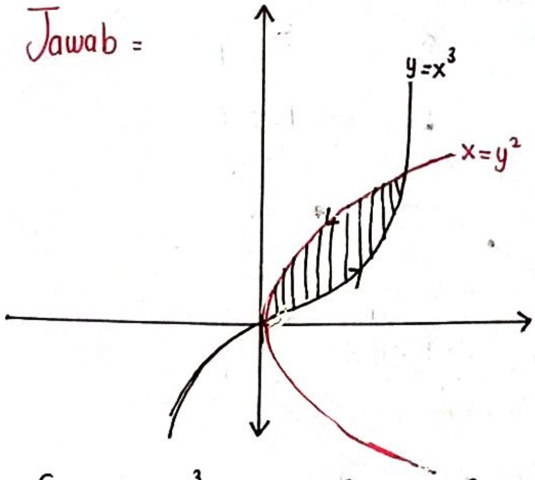
$$x^2y^3 + C_1 = y + x^2y^3 + C_2$$
$$C_1 = y + C_2$$

Jadi, diperoleh $\phi = x^2y^3 + y + C_2$

2). Diberikan $I = \oint_C (x-y)\,dx + (x+y)\,dy$ jika C kurva tertutup yang membatasi daerah $y = x^3$ dan $x = y^2$

(a). Hitung integral tersebut tanpa menggunakan Teorema Green

Jawab :



$C_1 : y = x^3$, $dy = 3x^2\,dx$ $C_2 : x = y^2$, $dx = 2y\,dy$

$$\int_C (x-y)\,dx + (x+y)\,dy = \int_{C_1} (x-y)\,dx + (x+y)\,dy + \int_{C_2} (x-y)\,dx + (x+y)\,dy$$

$$\Rightarrow \int_{C_1} (x-x^3)\,dx + (x+x^3)\,3x^2dx + \int_{C_2} (y^2-y)\,2y\,dy + (y^2+y)\,dy$$

$$= \int_0^1 (x-x^3)\,dx + (3x^3+3x^5)\,dx + \int_1^0 (2y^3-2y^2)\,dy + (y^2+y)\,dy$$

$$= \int_0^1 (x-x^3)\,dx + \int_0^1 (3x^3+3x^5)\,dx + \int_1^0 (2y^3-2y^2)\,dy + \int_1^0 (y^2+y)\,dy$$

$$= \tfrac{1}{2}x^2 - \tfrac{1}{4}x^4\Big|_0^1 + \tfrac{3}{4}x^4 + \tfrac{1}{2}x^6\Big|_0^1 + \tfrac{1}{2}y^4 - \tfrac{2}{3}y^3\Big|_1^0 + \tfrac{1}{3}y^3 + \tfrac{1}{2}y^2\Big|_1^0$$

$$= \cancel{\tfrac{1}{2}} - \tfrac{1}{4} + \tfrac{3}{4} + \cancel{\tfrac{1}{2}} - \cancel{\tfrac{1}{2}} + \tfrac{2}{3} - \tfrac{1}{3} - \cancel{\tfrac{1}{2}}$$

$$= \tfrac{1}{2} + \tfrac{1}{3}$$

$= \frac{5}{6}$ satuan luas

(b) Hitung integral tersebut dengan menggunakan teorema Green

$$\Rightarrow L = \iint \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\,dx\,dy$$

$$= \int_0^1 \int_{y^2}^{\sqrt[3]{y}} 1 - (-1)\,dx\,dy$$

$$= 2\int_0^1 x\Big|_{y^2}^{\sqrt[3]{y}}\,dy$$

$$= 2\int_0^1 y^{1/3} - y^2\,dy$$

$$= 2.\left[\tfrac{3}{4}y.\sqrt[3]{y} - \tfrac{1}{3}y^3\Big|_0^1\right]$$

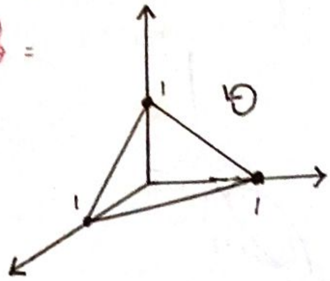
$$= 2\left[\tfrac{3}{4} - \tfrac{1}{3}\right]$$

$$= 2.\tfrac{5}{12}$$

$= \frac{5}{6}$ satuan luas

3). Gunakan teorema Stokes untuk menghitung $\oint F.dr$ dengan $F = xy\hat{i} + yz\hat{j} + 2x\hat{k}$ dan C adalah segitiga dengan titik sudut $(1,0,0), (0,1,0)$, dan $(0,0,1)$ terorientasikan searah dengan putaran jarum jam bila dilihat dari atas

Jawab :



$$\oint_c F.dr = \iint_c (\overline{\nabla} \times F).\overline{n}\, dS$$

- $F = (xy)\hat{i} + (yz)\hat{j} + (zx)\hat{k}$

$$\overline{\nabla} \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xy & yz & zx \end{vmatrix} = (0-y)\hat{i} - (z-0)\hat{j} + (0-x)\hat{k}$$
$$= -y\hat{i} - z\hat{j} - x\hat{k}$$

Diketahui persamaan bidang segitiganya diperoleh :

$$\phi : x + y + z = 1$$

$$\overline{n} = \frac{\overline{\nabla}\phi}{|\overline{\nabla}\phi|} = \frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{1+1+1}} = \frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$$

Karena dilihat dari atas, proyeksikan ke bidang XOY

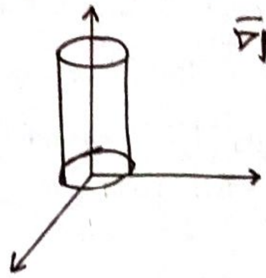
$$z = 1 - x - y$$
$$dS = \sqrt{(z_x)^2 + (z_y)^2 + 1}\; dx\, dy$$
$$= \sqrt{1+1+1}\; dx\, dy$$
$$= \sqrt{3}\, dx\, dy$$

Diperoleh :

$$\oint F.dr = \iint_s (\overline{\nabla} \times \overline{F}).\overline{n}\, dS$$
$$= \iint_s (-y\hat{i} - z\hat{j} - x\hat{k}).\frac{(\hat{i}+\hat{j}+\hat{k})}{\cancel{\sqrt{3}}}.\cancel{\sqrt{3}}\, dx\, dy$$
$$= \iint_s -y - z - x \;\; dx\, dy$$
$$= \iint_s -y - (1-x-y) - x \;\; dx\, dy$$
$$= \iint_s \cancel{-y} - 1 \cancel{+x} \cancel{+y} \cancel{-x}\; dx\, dy$$
$$= -\int_0^1 \int_0^{1-x} dy\, dx$$
$$= -\int_0^1 1 - x\, dx$$
$$= -\left[x - \tfrac{1}{2}x^2 \Big|_0^1\right]$$
$$= -\left[1 - \tfrac{1}{2}\right]$$
$$= -\tfrac{1}{2}$$

4). Hitung fluks medan vektor $\iint_S F.n\, dS$ jika diberikan vektor gaya $F = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ menembus permukaan tertutup S yang dibatasi oleh $x^2 + y^2 = 4$, $z = 0$, dan $z = 3$

Jawab :



$$\overline{\nabla} F = \left(\frac{\partial(4x)}{\partial x} + \frac{\partial(-2y^2)}{\partial y} + \frac{\partial z^2}{\partial z}\right)$$
$$= 4 - 4y + 2z$$

$x = r\cos\theta$
$y = r\sin\theta$

$$\iiint_s \overline{F}.\overline{n}\, dS = \iiint_V (\overline{\nabla}.F).\, dV$$
$$= \iiint_V (4 - 4y + 2z)\, dV$$
$$= \int_0^{2\pi}\int_0^2\int_0^3 (4 - 4r\sin\theta + 2z)\, r\, dz\, dr\, d\theta$$
$$= \int_0^{2\pi}\int_0^2 (4z - 4rz\sin\theta + z^2)\, r \Big|_0^3 dr\, d\theta$$
$$= \int_0^{2\pi}\int_0^2 (12 - 12r\sin\theta + 9)\, r\, dr\, d\theta$$
$$= \int_0^{2\pi}\int_0^2 12r + 9r - 12r^2\sin\theta\, dr\, d\theta$$
$$= \int_0^{2\pi}\int_0^2 21r - 12r^2\sin\theta\, dr\, d\theta$$
$$= \int_0^{2\pi} \frac{21}{2}r^2 - 4r^3\sin\theta \Big|_0^2$$
$$= \int_0^{2\pi} 42 - 32\sin\theta\, d\theta$$
$$= 42\theta + 32\cos\theta \Big|_0^{2\pi}$$
$$= 84\pi + \cancel{32(1)} - [0 + \cancel{32(1)}]$$
$$= 84\pi$$